Signal Processing Experiment

• Two parts
  – Investigate the Fourier spectrum of several periodic nonlinear signals
  – Investigate the artifacts of sampling (data acquisition)
Agilent Arbitrary Waveform Generator
Arbitrary Waveform Generator Control Panel

1. Graph Mode/Local Key
2. On/Off Switch
3. Modulation/Sweep/Burst Keys
4. State Storage Menu Key
5. Utility Menu Key
6. Help Menu Key
7. Menu Operation Softkeys
8. Waveform Selection Keys
9. Manual Trigger Key (used for Sweep and Burst only)
10. Output Enable/Disable Key
11. Knob
12. Cursor Keys
13. Sync Connector
14. Output Connector
Arbitrary Waveform Generator Rear Panel

1. External 10 MHz Reference Input Terminal (Option 001 only).
2. Internal 10 MHz Reference Output Terminal (Option 001 only).
3. External Modulation Input Terminal
4. Input: External Trig/FSK/Burst Gate
   Output: Trigger Output
5. USB Interface Connector
6. LAN Interface Connector
7. GPIB Interface Connector
8. Chassis Ground
Programming

• Use LabVIEW program to take analog signals from the Arbitrary Waveform Generator
• Display analog data vs. time (compare to oscilloscope)
• Display analog data vs. frequency (program spectrum analyzer)

• Open Intuilink software to communicate with the Arbitrary Waveform Generator
• Program generator waveform
• Start Generator (continuous waveform)
• Start data acquisition
Fourier Spectrum

Investigate spectrum of non-linear waveforms using Fast Fourier Transform (FFT) in LabVIEW

1. Investigate spectrum of pure sine wave
2. Investigate popular nonlinear waveforms – square wave, pulse train, triangle wave (compare results to theory)
3. Investigate typical modulated waveforms – amplitude modulation (AM) & frequency modulation (FM) (compare results to theory)
4. Investigate the effect of adding bias (offset) to signals
5. Investigate spectrum of arbitrary nonlinear signal
Different Waveforms

• Look up Fourier Spectrum for these signals
  – Single frequency sine wave
  – Square wave
  – Triangle wave
  – Pulse Waveform (non symmetric square wave)
• Fourier Spectrum of arbitrary waveform
• Effects of adding offset
Single frequency sine wave

Square wave

Triangle wave

Pulse Waveform

Effects of adding offset
Arbitrary Waveform

• Free-hand draw some arbitrary waveform

• Provide the waveform (vs. time) and the spectrum (vs. frequency) in report
Programming the
Arbitrary Waveform Generator
Programming the Arbitrary Waveform Generator
Select Generator

![Select Generator Diagram]
Use Waveform Editor to Draw Arbitrary Waveform
Use Waveform Math to Draw Arbitrary Waveform
Sending Waveform to Generator
Use Equation Calculator to Draw Standard Waveforms (Sine, Square, Triangle) & Modulated Waveforms
Amplitude Modulated (AM) Waveform
Sine Waves at 2 Frequencies
Frequency Modulated (FM) Waveform
Modulated Waveforms

• Single frequency tone modulation in both cases
  – Amplitude modulated signal (AM wave)
  – Frequency modulated signal (FM wave)

• Vary tone frequency keeping modulation constant
AM wave

Radio frequency (RF)

Audio frequency (AF)

Amplitude modulated RF carrier
AM spectrum

\[
\text{Lower Sideband: } \frac{A_0}{2} \sum_{m=1}^{M} a_m \cos \left(2\pi (f_m - f_c) - \varphi_m \right)
\]

\[
\text{Upper Sideband: } \frac{A_0}{2} \sum_{m=1}^{M} a_m \cos \left(2\pi (f_m + f_c) + \varphi_m \right)
\]

\[
\text{Carrier: } A_0 \cos \left(2\pi f_c t \right)
\]

\[
\text{Modulation: } m \{ t \}
\]

\[
0 \quad f_{\text{max}} \quad f_c - f_{\text{max}} \quad f_c \quad f_c + f_{\text{max}}
\]

\[
B_M \quad \text{Info. bandwidth} \quad \text{Frequency} \quad B_T \quad \text{Transmission bandwidth}
\]

Figure 9.4 Spectra of modulation & AM wave.
FM wave
FM spectrum

<table>
<thead>
<tr>
<th>Ref, 0=1.1V, 1=1.4V</th>
<th>Modulation Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 kHz</td>
<td>5 MHz</td>
</tr>
<tr>
<td>2 MHz</td>
<td>10 MHz</td>
</tr>
</tbody>
</table>

| 930 MHz             | 970 MHz               |
Part 2 – Sampling Artifacts

• Signal-to-Noise Ratio
  – Signal Detection
  – Effects of Quantization

• Discrimination & Dynamic Range (sample size)

• Aliasing & Bandwidth (sampling frequency)
Why Does Spectrum of Pure Sine Wave Appear to Have Noise Associated with It?

![Graph showing amplitude vs frequency with noise](image_url)
Sampling in Time

As sampling frequency increases, the digital signal better approximates the analog signal.
Quantization in Amplitude

As resolution increases, the digital signal better approximates the analog signal.
The maximum quantization error is $\frac{1}{2}$ the increment size.
Quantization Noise

If you assume that the noise is uniformly distributed & uncorrelated with the signal then the rms (root mean square) value of the noise is

\[
\frac{1}{\sqrt{12}} \text{ LSB}
\]

(the standard deviation of the uniform distribution)
Quantization Noise 16 bit Converter

**Green** – analog signal

**Red** – D/A output

**Blue** - noise
Sampling Effects

• Single frequency sine wave with white noise (Signal-to-Noise (S/N) ratio)
  – Add White Noise to single frequency until noise obscures the signal
  – Change sampling frequency & note effect

• Frequency discrimination versus sample size
  – Produce Sine waves at 2 very closely spaced frequencies
  – Change sample size until both frequencies appear in spectrum

• Aliasing
Aliasing

- Nyquist Theorem – sample at greater than twice the rate of the maximum frequency component in the input signal

- Nyquist frequency – half the sampling frequency

- Only possible to recover frequencies at or below Nyquist (so what is the bandwidth of the data acquisition system?)

- Frequencies above Nyquist will “alias” between DC and Nyquist

- Alias frequency = Absolute Value (closest integer multiple of sampling frequency - input frequency)
Aliasing Example

Sampling Frequency $f_s=100$

$Nyquist Frequency = \frac{f_s}{2} = 50$

F1 25Hz
F2 70Hz
F3 160Hz
F4 510Hz

$F_2 = 30$ Hz
$F_3 = 40$ Hz
$F_4 = 10$ Hz

Red
Black
Arrow – Actual Frequency
Arrows – Alias

F4 alias 10Hz
F1 25Hz
F2 alias 30Hz
F3 alias 40Hz
Preventing Aliasing

- Oversampling
- Use filters designed to eliminate frequencies over Nyquist frequency
- Use combination of both because real-world filters exhibit some roll-off
- Use special antialiasing filters that have sharp roll-off

\[ \begin{align*}
V_{out} & \quad V_{in} \\
0.0 & \quad 0.0 \\
1.0 & \quad 1.0
\end{align*} \]

Ideal Filter

\[ \begin{align*}
V_{out} & \quad V_{in} \\
0.0 & \quad 0.0 \\
1.0 & \downarrow
\end{align*} \]

Real-World Filter

\[ \begin{align*}
V_{out} & \quad V_{in} \\
0.0 & \quad 0.0 \\
1.0 & \downarrow
\end{align*} \]
Why is Signal Processing Important for Mechanical Engineering?

Comment on this in report.

Using Spectrum Analysis & Signal Analysis Techniques to Determine Faults in Motors & Motor Driven Systems
Vibration Signature Analysis

• **What is Vibration Signature Analysis?**
  – Measure the vibration characteristic of a system as a function of time.
  – Take the frequency response of the signal.
  – Correlate a specific frequency component or set of frequency components with a failure of the system to predict the failure before it occurs.
What is Vibration?

• **Vibration**: Any motion that repeats itself after an interval of time is called a vibration or oscillation.

• The swinging of a pendulum and the motion of a plucked string are typical examples of vibration.

• For a motor driven system, the vibration characteristic can give an indication of impending system failure.
Now Consider the Situation Where the Force on a Housing is Caused by the Rotating Unbalance of a Motor in the Housing

Machine of total mass $m$ i.e. $m_0$ is included in $m$

$e = \text{eccentricity}$

$m_o = \text{mass unbalance}$

$\omega_r = \text{rotation frequency}$

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What force is imparted on the structure? Note it rotates with x component:

\[ x_r = e \sin \omega_r t \]
\[ \Rightarrow a_x = \ddot{x}_r = -e \omega_r^2 \sin \omega_r t \]

From dynamics,

\[ R_x = m_0 a_x = -m_0 e \omega_r^2 \sin \theta = -m_0 e \omega_r^2 \sin \omega_r t \]
\[ R_y = m_0 a_y = -m_0 e \omega_r^2 \cos \theta = -m_0 e \omega_r^2 \cos \omega_r t \]
The problem is now just like the situation of a Single Degree of Freedom (SDOF) system with a harmonic excitation

\[ m\ddot{x} + c\dot{x} + kx = m_0 e\omega_r^2 \sin \omega_r t \]

or

\[ \ddot{x} + 2\xi \omega_n \dot{x} + \omega_n^2 x = \frac{m_0}{m} e\omega_r^2 \sin \omega_r t \]

Note the influences on the forcing function

(we are assuming that the mass \( m \) is held in place in the \( y \) direction as indicated in the figure)
• Just another SDOF oscillator with a harmonic forcing function
• Expressed in terms of frequency ratio $r$

\[ x(t) = X \sin(\omega_r t - \phi) \]

\[ X = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \]

\[ \phi = \tan^{-1}\left( \frac{2\zeta r}{1 - r^2} \right) \]
Displacement magnitude vs frequency caused by rotating unbalance
Transducers for Vibration Signature Analysis

- Displacement Transducer
- Velocity transducers
- Piezo-Electric Accelerometer

Using Displacement Transducers to measure more than one degree of freedom
• **Displacement Transducers**
  – Measures the displacement of the housing
  – Non-contact type are typically capacitive or inductive (eddy current) essentially measuring the change in air gap of a capacitor or magnetic circuit
  – Contact type can be resistive or LVDTs (Linear Variable Differential Transformer)
  – Capacitive, inductive and LVDT require electronic processing of the signals
  – Good for measuring low frequency vibrations
  – Very limited output for low amplitude high frequency vibrations
**Accelerometers**

- Measures the acceleration of the housing
- Piezoelectric type measures the force produced by an accelerating mass (seismic mass) – cannot measure DC acceleration
- MEMS type often measure the displacement of the seismic mass mounted on a cantilever beam using a non-contact (capacitive or inductive) displacement transducer – can be used to measure DC
- Requires electronic processing of the signals
- Good for measuring high frequency vibrations
- Very limited output for low frequency vibrations (unless high amplitude)
• **Velocity Transducers**
  – Measures the velocity of the housing
  – Essentially moving coil or moving magnet linear tachometers
  – May not require any additional electronic processing of the signals
  – Good for measuring both low & high frequency vibrations
Velocity & Acceleration are Related to Displacement

\[ y(t) = A \sin (\omega t + \phi) = A \cos (\omega t + \phi - \frac{\pi}{2}) \]

\[ \dot{y}(t) = \frac{d}{dt} y(t) = A \omega \cos (\omega t + \phi) = A \omega \sin (\omega t + \phi + \frac{\pi}{2}) \]

\[ \ddot{y}(t) = \frac{d^2}{dt^2} y(t) = -A \omega^2 \sin (\omega t + \phi) = A \omega^2 \sin (\omega t + \phi + \pi) \]
Displacement, Velocity & Acceleration for Simple Harmonic Motion

\[ y(t) = 1.0 \sin \left( 2t + \frac{\pi}{2} \right) \]
Transducer Usefulness Vs. Frequency

- **Relative Amplitude**
  - 10^8:1
  - 10^6:1
  - 10^4:1
  - 100:1
  - 1

- **Frequency**
  - 0.2
  - 2
  - 20
  - 200
  - 2k
  - 20kHz

- **Transducer Types**
  - Piezoelectric Accelerometer
  - Eddy Current Proximity Probe
  - Velocity Transducer
Vibration Signature Analysis Process

Unbalance mass

Bearing fault

Gear revolution

Frequency domain

We can divided the frequency component from mechanical elements

FFT Analyzer
• **Air Gap Eccentricity**

**In 3 degrees of Freedom**

![Diagram showing air gap eccentricity in 3 degrees of freedom with labels for parallel misalignment, angular misalignment, and combined misalignment.](image)
Rotor Static Unbalance

**Static Unbalance**
- Equal phase on each bearing
- Mainly radial vibration

**Coupled Unbalance**
- Phase changes 180° across bearing
- Mainly radial vibration

**Overhung Rotor Unbalance**
- Both radial and horizontal vibration
- Often both static and dynamic unbalance are seen together

Please Note: Strong unbalance cause harmonics
Rotor Unbalance

Static Unbalance

Couple Unbalance

Dynamic Unbalance

F

F_1

F_2

C_g

C_g

e